THE USE OF FUZZY MEASURES IN ESTIMATING THE INTERDEPENDENCIES BETWEEN ACADEMIC SUBJECTS

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ABSTRACT. Computing the overall student's academic achievements is often done by adding all the scores of the student and dividing the value by the number of subjects. This action means that all subjects are seen as having the same weights or same relative importance. Besides that, the subjects are assumed to be independent with each other which is not true in the real context. This paper focuses on estimating the interactions or interdependencies measures among five main subjects taught in Malaysian primary schools where a result from one case study was used as a basis of the discussion. Five experienced teachers were asked to weigh the importance of the subjects by direct rating method. The results showed that the teachers gave same weight to two subjects, whereas the other three subjects have different weights. Furthermore, one mathematical technique known as lamda (λ) – fuzzy measure was used to compute the interdependencies between the selected subjects. The highest interdependency occurred in pair of mathematics and science subjects and the lowest interaction or interdependency was between Malay Language subjects and English. Even though this result cannot be generalized and is not strange to researchers in the field but this study has succeeded in quantifying the interdependencies among the selected subjects. In future research, the relative weights as well as the interdependencies values are suggested to be considered in computing the overall scores of students' academic achievements. By doing so, the overall results would explain the real performance of the students.

Keywords: fuzzy measures, interdependencies, weights, academic subjects, performance

INTRODUCTION

It is a normal practice for teachers or schools in general to monitor students' academic performance by conducting assessments such as examinations or tests. The results from these assessments were used to review students' progress. The final results of the students are often represented as single scores where certain average methods were used to obtain these composite scores. If simple average method were used, all scores obtained by each student were added together, and this total score would be divided by the number of related subjects. This action means that all the subjects are treated equally and all of these subjects are having the same relative degree of difficulty or importance. Besides that, it implies that the subjects are independent with each other. Furthermore, students' achievement in certain academic subjects was assumed to be unrelated with their achievement in other subjects, which is naturally and logically not true all the time. Hence, the use of these final results for further monitoring purposes may be misleading.

This paper is introducing two measures: the relative importance and the interactions or interdependencies among the academic subjects. The direct rating method (Wagholikar, 2007; Wagholikar& Deer, 2007; Wagholikar& Jo, 2008) was used to estimate the first measure, whereas the λ-fuzzy measure was used to estimate the interaction measures among five main academic subjects in primary schools in Malaysia. This interaction concept was introduced by Sugeno in 1974 and often used to solve many multi-criteria problems in many real life situations (Grabisch & Roubens, 2008). This paper demonstrates the use of the two methods in a case study conducted in Malaysia. The discussion focuses on the relative importance of the academic subjects and the interdependencies in each pair and in group of three of the subjects.

CASE STUDY

Majority of primary school pupils in Malaysia are in the age of six to 12 years. Currently, at the end of year six, pupils would take a national examination which is known as Primary School Evaluation Test (PSET). This test consists of five main subjects: Mathematics (M), Science (S), Malay Language Comprehension (MC), Malay Language Written (MW) and English Language (E). The test is conducted by the Malaysian Examination Council, and the tests' results are usually used as a basis for entrance to boarding schools in Malaysia. In the process of monitoring students' performance towards PSET, students would take a series of tests at school or state levels. The scores of the tests are in the range of zero to 100. Besides checking the grades that the students might receive from these tests, the schools usually calculate the overall scores by taking simple average, that is by adding all the scores and divide the total with five (5), that is the total number of subjects. The students were ranked based on these overall scores. In this small case study, five experienced teachers who taught the year-six students in a primary school in Perlis were selected as the respondents. These five teachers were teaching the five respective academic subjects for more than five years when the case study was conducted.

METHODS

A short survey was done by asking these five experienced teachers to rate the degree of importance of these five subjects by direct rating method as illustrated in the Table 1. After the relative importance or the fuzzy measure weights of the academic subjects was determined, the degree of interactions or the fuzzy measures among the subjects could be quantified.

In this study, λ-fuzzy measure (Marichal, 1999) was used to determine amount of interactions among the subjects. There is a parameter, λ, which describes the degree of additivity between subjects and represents the membership degree between the subjects such as independencies, complementaries, and redundancies. Fuzzy measures were also called as monotone measures or non-additive measures or capacities constitute a generalization of classical probability distributions where additivity is removed and monotonicity is imposed instead.

Table 1. Linguistic scales for the importance weight

<table>
<tr><td rowspan="5">The subject is less important</td><td>Extremely</td><td>0.0</td></tr>
<tr><td>Highly</td><td>0.1</td></tr>
<tr><td>Very</td><td>0.2</td></tr>
<tr><td>Strongly</td><td>0.3</td></tr>
<tr><td>Moderately</td><td>0.4</td></tr>
<tr><td></td><td>Equally</td><td>0.5</td></tr>
</table>

The subject is more important	Moderately	0.6
	Strongly	0.7
	Very	0.8
	Highly	0.9
	Extremely	1.0

Let $X = \{X_1, \dots, X_n\}$ be the set of academic subjects and $P(X)$ denotes the power set of X or set of all subsets of X. A fuzzy measure μ on the set X of subjects is a set function $\mu: P(X) \rightarrow [0,1]$ satisfying certain properties such as boundary condition, monotonicity, super-additive and sub-additive (Ishii & Sugeno, 1985). However, since μ is non-additive in general, it is necessary to define the 2^n coefficients corresponding to the 2^n subsets of X. The fuzzy measure of a finite set can be obtained from the set of values of the fuzzy measure weights $\mu_i = \mu_\lambda(\{X_i\})$ for $i = 1, \dots, n$ which is formulated as the following equation when $\lambda \neq 0$ and $-1 < \lambda < \infty$ (Leszczyński, Penczek & Groculkski, 1985).

$$\mu_\lambda(\{X_1, \dots, X_n\}) = \frac{1}{\lambda} \left| \prod_{i=1}^{n} (1 + \lambda \cdot \mu_i) - 1 \right| \quad (1)$$

Based on Eq. 1, and since the boundary conditions, $\mu_\lambda(X) = 1$, the parameter λ can be calculated by solving the following equation

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda \cdot \mu_i) \quad (2)$$

Moreover, μ is non-additive in general, so, it is necessary to define the 2^n coefficients corresponding to the 2^n subsets of X.

RESULTS

Relative importance of the selected academic subjects

The judgment of the relative importance or the fuzzy measure weights of the five subjects by the five experts is summarized as in Table 2.

Table 2. Judgment of the relative importance of five subjects by five experts

Subjects	Experts				
	T1	T2	T3	T4	T5
M	0.50	0.50	0.50	0.60	0.50
S	0.50	0.50	0.60	0.60	0.50
MC	0.30	0.40	0.50	0.30	0.30
MW	0.30	0.30	0.40	0.40	0.40
E	0.30	0.30	0.40	0.30	0.40

Based on Table 3, expert T1 evaluated M and S as having about the same relatively higher importance than the other three subjects. The same trend is found in the other four judgments. All evaluators seemed to judge M and S subjects as more important than the other three language subjects**.** The arithmetic average for each subject is summarized in Table 3.

Table 3. The average weight of importance of the five subjects

Subjects	Weights
M	0.52
S	0.54
MC	0.36
MW	0.36
E	0.34

Based on Table 3, S subject receives the highest score; M subject is at the second highest position with only 0.2 different from S subject. This shows that the selected teachers evaluated sciences subjects as the most important subjects but chose M to be a little less important than S. This might be due to their observation on how their pupils studied and struggled to have good grades for these two subjects. Besides that, it is well known that science subjects are more difficult to master as compared to other subjects. The subjects related to Malay language, MC and MW shared the third ranking, and E subject is at the last ranking. This implies that the two Malay language subjects share the same degree of importance or difficulty but English was seen a little less important than the Malay language subjects.

Interdependencies among the selected subjects

Thus, we proceed to apply Eq. 1 once the fuzzy measure weight for each subject is identified. However, we need to provide the λ value which describes the degree of additivity between the subjects before applying Eq. 1. The value of λ can be calculated by equation (2). The calculation is shown as follows.

$$\lambda+1=(1+\lambda\cdot\mu_M)(1+\lambda\cdot\mu_S)(1+\lambda\cdot\mu_{MC})(1+\lambda\cdot\mu_{MW})(1+\lambda\cdot\mu_E)$$

$$\lambda+1=(1+0.52\lambda)(1+0.54\lambda)(1+0.36\lambda)(1+0.36\lambda)(1+0.34\lambda)$$

By using the Mathematica 5.2 Kernel Software,

$$\lambda+1=1+2.12\lambda+1.7788\lambda^2+0.738576\lambda^3+0.151839\lambda^4+0.0123732\lambda^5$$

$$F(\lambda)=1.12\lambda+1.7788\lambda^2+0.738576\lambda^3+0.151839\lambda^4+0.0123732\lambda^5$$

where $F(\lambda)=0$

Therefore, the roots of λ are

$$\lambda=\{-5.77738,\ -2.78762-3.04583i,\ -2.78762+3.04583i,\ -0.9190,\ 0\}$$

Since $-1<\lambda<\infty$, the accepted values of λ are – 0.9190 and 0. The λ value is negative, this implies the existence of substitutive effect or redundancy between the subjects. However, λ = 0 is an additive measure which signifies that there is no interaction between the attributes and it is impossible to be taken into account in obtaining the fuzzy measure. Furthermore, since μ_λ is non-additive in general, and there were five academic subjects, so, it is necessary to define the 2^5 fuzzy measure coefficients corresponding to the 32 subsets of subjects. This paper only highlights on the interdependencies measures between two subjects and among three subjects as summarized in Table 4.The calculation of interdependency measure between MC, and MW subjects is given as follows.

Table 4. The interdependencies measures between 2 or 3 academic subjects

2 academic subjects	Inter dependencies measures	3 Academic subjects	Inter dependencies measures
MC, MW	0.60	MC, MW, E	0.75
MC, E	0.59	MC, MW, M	0.83
MC, M	0.71	MC, MW, S	0.84
MC, S	0.72	MC, E, M	0.83
MW,E	0.59	MC, E, S	0.84
MW, M	0.71	MC, M, S	0.90
MW, S	0.72	MW, E, M	0.83
E, M	0.70	MC, E, S	0.84
E, S	0.71	MC, M,S	0.90
M, S	0.80	E, M, S	0.90

Based on Table 4, the pair of M and S received the highest interdependency measure or combined importance of two subjects (Wang, Lee & Tzeng, 2005) whereas the pair of MC and E had the least degree of interaction between two subjects. The last column of Table 4 shows the estimated measures of interdependencies among three subjects. Each of the three sets of subjects which contain M and S received the equal highest interaction values, 0.90, whereas the set that contains all language subjects, MC, MW and E received the lowest measure of interaction.

CONCLUSION

This paper introduces the measures of relative importance and interdependencies between five main academic subjects in the context of primary schools in Malaysia. Two methods were used to estimate the two measures. The results from the case study conducted show that as individual subjects, Science was given the highest importance, and English was the least important subject. In terms of interactions between subjects, Mathematics and Science had the highest interaction, and the interactions or interdependencies are higher between three subjects when Mathematics and Science subjects were two out of the three subjects. These measures of different relative importance of the subjects and/or the degree of interdependencies among the subjects must be considered in calculating the overall scores of students' academic achievements (Maznah & Siti Rohana, 2013). By doing so, the overall scores could reflect the true performance of the students. Besides that, these two concepts should be considered in other real problems so that the true setting of the problems is still preserved especially when some kind of evaluations is used in those studies.

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